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Conformal Change of Douglas Space of Second Kind with Certain Special $(\alpha, \beta) -$ Metric

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Abstract: - In year 2008 IL- Yong Lee defines a Douglas space of second kind of a Finsler space with an $(\alpha, \beta) -$ metric. In year 2015, Gauree Shanker and Deepti Choudhary [1] find the conditions under which the conformal change of Finsler space with Matsumoto and generalized Kropina metric is of Douglas space of second kind. Further, In the continuing the study of Douglas space of second kind we find the conditions under which the conformal change of Finsler space with Special $(\alpha, \beta) -$ metric is of Douglas space of second kind.
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Index Terms: - conformal change, Douglas space, Douglas space of second kind, Special $(\alpha, \beta) -$ metric.

1. INTRODUCTION:-

A Finsler space is said to be of Douglas type or a Douglas space if $D^{ij} = G^i y^j - G^j y^i$ are homogeneous polynomial in (y^i) of degree 3.The notion of Douglas space was introduced by S. Bacso and M. Matsumoto [7] as a generalization of Berwald space from viewpoint of geodesic equations. The notion of weakly-Berwald space as another generalization of Berwald space was introduced by S. Bacso and B.Szilagyi [6]. Recently, I. Y. Lee [9] has studied Douglas space of second kind and he has find the conditions for a Finsler space with Matsumoto metric to be a Douglas space of second kind. It is remarkable that a Finsler space is a Douglas space if and only if the Douglas tensor $D_{i\ jk}^{h}$ vanishes identically [9]. The theories of Finsler spaces with an $(\alpha, \beta) -$ metric have contributed to the development of Finsler geometry [14], and Berwald spaces with an $(\alpha, \beta) -$ metric have been treated by some authors ([2], [13], [17]).

A Finsler space F^n is said to be a Douglas space if $D^{ij} = G^i(x, y)y^j - G^j(x, y)y^i$ are homogeneous polynomials in (y^i) of degree three. Then a Finsler space F^n is said to be a Douglas space of the second kind if and only if $D^{im}{}_m = (n + 1)G^i - G^m{}_m y^i$ are homogeneous polynomials in (y^i) of degree two. On the other hand, in [15] a Finsler space with a $(\alpha, \beta) -$ metric is a Douglas space if and only if $B^{ij} = B^i y^j - B^j y^i$ are homogeneous polynomials in (y^i) of degree three. Then a Finsler space of a $(\alpha, \beta) -$ metric is said to be a Douglas space of the second kind if and only if $B^{im}{}_m = (n + 1)B^i - B^m{}_m y^i$ are homogeneous polynomials in (y^i) of degree two, where $G^m{}_m$ is given by [11].

The conformal theory of Finsler space was introduced by M. S. Kneblman in 1929 [7] and it has been investigated in detail by M. Hashiguchi, [6]. Later on Y. D. Lee [10] and B. N. Prasad [16] found conformally invariant tensors in the Finsler space with $(\alpha, \beta) -$ metric. In [14], conformal transformation of Douglas space with special $(\alpha, \beta) -$metric have been studied by S. K. Narasimhamurthy.

The purpose of the present paper is to find the conditions for a Douglas space of second kind with $(\alpha, \beta) -$metric to be a Douglas space of second kind under conformal transformation. Further we have proved that Douglas space of second kind with special $(\alpha, \beta) -$ metric is a Douglas space of second kind under conformal transformation.

2. PRELIMINARIES:-

Let $F^n = (M^n L(\alpha, \beta))$ be an n-dimensional Finsler space, where M is a differential manifold of dimension n and $L(x, y)$ $(where\, y^i = \dot{x}^i)$ is the fundamental function defined on the slit tangent bundle $TM_0 = TM \setminus \{0\}$ of manifold M. We assume that $L(x, y)$ is positive and the metric tensor $g_{ij}(x) = \frac{1}{2} \dot{\partial}_i \dot{\partial}_j L^2$ is positive definite, where $\dot{\partial}_i = \frac{\partial}{\partial y^i}$.

The geodesics of an n-dimensional Finsler space $F^n = (M^n L)$ are given by the system of differential equations [6]

$$\frac{d^2x^i}{dt^2}y^j - \frac{d^2x^j}{dt^2}y^i + 2(G^i x^j - G^j x^i) = 0 \ , y^i = \frac{dx^i}{dt}$$

in parameter t.

The function $G^i(x,y)$ is given by

$$2G^i(x,y) = g^{ij}\left(y^r \dot{\partial}_j \partial_r F - \partial_j F\right) = \gamma^i_{jk} y^j y^k$$

where $\dot{\partial}_i = \frac{\partial}{\partial y^i}$, $F = \frac{L^2}{2}$, γ^i_{jk} are Christoffel symbols constructed from $g_{ij}(x,y)$ with respect to x^i and $g^{ij}(x,y)$ is the inverse of fundamental metric tensor $g_{ij}(x,y)$.

In [2], It has been shown that F^n is a Douglas space if and only if the Douglas tensor [5]

$$D^i_{j\,k} = G^i_{j\,k} - \frac{1}{n+1}\left(G_{ijk}y^h + G_{ij}\delta^h_k + G_{ik}\delta^h_j + G_{jk}\delta^h_i\right)$$

vanishes identically, where $G^h_{i\,j\,k} = \dot{\partial} G^h_{i\,j}$ is the hv-curvature tensor of the Berwald connection $B\Gamma$[12].

The F^n is said to be a Douglas space [4] if

$$D^{ij} = G^i(x,y)y^j - G^j(x,y)y^i \qquad (2.1)$$

are homogeneous polynomials in (y^i) of degree three. Differentiating (2.1) with respect to by y^h, y^k, y^p and y^q, we have $D^{ij}_{hkpq} = 0$, which are equivalent of $D^{im}_{hkpm} = (n+1)D^i_{h\,kp} = 0$. Thus if a Finsler space F^n satisfies the condition $D^{ij}_{hkpq} = 0$, which are equivalent to $D^{im}_{hkpm} = (n+1)D^i_{h\,kp} = 0$, we call it a Douglas space. Further differentiating (2.1) by y^m and contacting m and j in the obtained equation, we have $D^{im}{}_m = (n+1)G^i - G^m{}_m y^i$. Thus F^n is said to be a *Douglas space of the second kind* if and only if

$$D^{im}{}_m = (n+1)G^i - G^m{}_m y^i \qquad (2.2)$$

are homogeneous polynomials in (y^i) of degree two. Furthermore differentiating (2.2) with respect to y^h, y^j and y^k, we get $D^{im}_{hjkm} = (n+1)D^i_{h\,jk} = 0$. Therefore we have

Definition 2.1:- If a Finsler space F^n satisfies the condition that $D^{im}{}_m = (n+1)G^i - G^m{}_m y^i$. be homogeneous polynomials in (y^i) of degree two, we call it a *Douglas space of the second kind*.

On the other hand, a Finsler space of an $(\alpha,\beta)-$ metric is said to be a *Douglas space of the second kind* if and only if

$$B^{im}{}_m = (n+1)B^i - B^m{}_m y^i.$$

are homogeneous polynomials in (y^i) of degree two, where $B^m{}_m$ is given by [10]

A Finsler metric $L(x,y)$ is called an $(\alpha,\beta)-$ metric, if L is a positively homogeneous function $L(\alpha,\beta)$ of degree one in two variables $\alpha(x,y) = \sqrt{a_{ij}(x)y^i y^j}$ and $\beta = b_i(x)y^i$. The space $R^n = (M,\alpha)$ is called the associated Riemannian space with F^n. We use the following symbols [12]:

$$b^i = a^{ir}b_r, \quad b^2 = a^{rs}b_r b_s, \quad 2r_{ij} = b_{i;j} + b_{j;i},$$
$$r^i_j = a^{ir}r_{ij}, \quad s^i_j = a^{ir}s_{rj}, \quad r_i = b_r r^r_i,$$
$$2s_{ij} = b_{i;j} - b_{j;i}, \quad s_i = b_r s^r_i.$$

Further, a Finsler space with an $(\alpha,\beta)-$ metric is said to be a Douglas space of the second kind if and only if

$$B^{im}{}_m = (n+1)B^i - B^m{}_m y^i.$$

are homogeneous polynomials in (y^i) of degree two, where $B^m{}_m$ is given by [9]. Furthermore differentiating the above with respect to y^h, y^j and y^k, we get $B^{im}_{hjkm} = B^i_{h\,jk} = 0$.

Thus, we have

Definition 2.2.:- A Finsler space of an $(\alpha,\beta)-$ metric is said to be a *Douglas space of the second kind* if and only if it satisfies the condition that $B^{im}{}_m = (n+1)B^i - B^m{}_m y^i$ are homogeneous polynomials in (y^i) of degree two.

Since $L = L(\alpha,\beta)$ is a positively homogeneous function of α and β of degree one, we have

$$\left.\begin{aligned} &L_\alpha \alpha + L_\beta \beta = L, \qquad L_{\alpha\alpha}\alpha + L_{\alpha\beta}\beta = 0 \\ &L_{\beta\alpha}\alpha + L_{\beta\beta}\beta = 0, \quad L_{\alpha\alpha\alpha}\alpha + L_{\alpha\alpha\beta}\beta = -L_{\alpha\alpha} \\ &L_\alpha = \frac{\partial L}{\partial \alpha}, \quad L_\beta = \frac{\partial L}{\partial \beta}, \quad L_{\alpha\alpha} = \frac{\partial^2 L}{\partial\alpha\partial\alpha} \\ &L_{\alpha\beta} = L_{\beta\alpha} = \frac{\partial^2 L}{\partial\alpha\partial\beta}, \quad L_{\alpha\alpha\alpha} = \frac{\partial^3 L}{\partial\alpha\partial\alpha\partial\alpha} \end{aligned}\right\} \qquad (2.3)$$

Here we state the following lemma and remark for the later frequent use:

Remark 2.3:- Throughout the present paper, we say "homogeneous polynomial(s) in (y^i) of degree r" as hp(r) for brevity. Thus $\gamma^i_{0\,0}$ is hp(2) and, if the Finsler space with an $(\alpha,\beta)-$ metric is a Douglas space of the second kind, then $B^{im}{}_m$ is hp(2).

Theorem 2.4[3]:- The necessary and sufficient condition for a Finsler space F^n with an $(\alpha,\beta)-$ metric to be a Douglas space of the second kind is that $B^{im}{}_m$ are homogeneous polynomials in (y^m) of degree two, where $B^{im}{}_m$ is given by (2.4) and (2.5), provided that $\Omega \neq 0$.

$$B^{im}{}_m = \frac{(n+1)\alpha L_\beta}{L_\alpha} s^i{}_0 + \frac{\alpha\{(n+1)\alpha^2 \Omega L_{\alpha\alpha} b^i + \beta\gamma^2 A y^i\}}{2\Omega^2} r_{00} - \frac{\alpha^2\{(n+1)\alpha^2 \Omega L_\beta L_{\alpha\alpha} b^i + B y^i\}}{L_\alpha \Omega^2} s_0 - \frac{\alpha^3 L_{\alpha\alpha} y^i}{\Omega} r_0 \quad (2.4)$$

where

$$\left.\begin{array}{c} A = \alpha L_\alpha L_{\alpha\alpha\alpha} + 3L_\alpha L_{\alpha\alpha} - 3\alpha(L_{\alpha\alpha})^2 \\ B = \alpha\beta\gamma^2 L_\alpha L_\beta L_{\alpha\alpha\alpha} + \\ \beta\{(3\gamma^2 - \beta^2)L_\alpha - 4\alpha\gamma^2 L_{\alpha\alpha}\}L_\beta L_{\alpha\alpha} + \Omega L L_{\alpha\alpha} \\ \Omega = (\beta^2 L_\alpha + \alpha\gamma^2 L_{\alpha\alpha}) \end{array}\right\} \quad (2.5)$$

3. CONFORMAL CHANGE OF DOUGLAS SPACE OF SECOND KIND WITH $(\alpha,\beta)-$ METRIC:-

In [1], the condition on conformal change, so that a Douglas space of second kind is conformally transformed to a Douglas space of second kind is such as:-

Let $F^n = (M, L)$ and $\bar{F}^n = (M, \bar{L})$ be two Finsler space on the same underlying manifold M. If we have a function $\sigma(x)$ in each coordinate neighborhood of M such that $\bar{L}(x,y) = e^\sigma L(x,y)$, then F^n is called conformal to $\bar{F}^n$ and the change $L \to \bar{L}$ of metric is called conformal change. As to $(\alpha,\beta)-$metric, $\bar{L} = e^\sigma L(\alpha,\beta)$ is equivalent to $\bar{L}(x,y) = L(e^\sigma \alpha, e^\sigma \beta)$ by homogeneity. Therefore, a conformal change of $(\alpha,\beta)-$metric is expressed as $(\alpha,\beta) \to (\bar{\alpha},\bar{\beta})$ where $\bar{\alpha} = e^\sigma \alpha$, $\bar{\beta} = e^\sigma \beta$. Therefore, we have $\bar{a}_{ij} = e^{2\sigma} a_{ij}$, $\bar{b}_i = e^\sigma b_i$ therefore, we have

$$\left.\begin{array}{c} \bar{a}_{ij} = e^{2\sigma} a_{ij}, \bar{b}_i = e^\sigma b_i \\ \bar{a}^{ij} = e^{*2\sigma} a^{ij}, \quad \bar{b}^i = e^{*\sigma} b^i \end{array}\right\} \quad (3.1)$$

and $b^2 = a^{ij} b_i b_j = \bar{a}^{ij} \bar{b}_i \bar{b}_j$. Thus we state the following.

Proposition 3.1:-. A Finsler space with $(\alpha,\beta)-$metric and the length b of b_i with respect to the Riemannian metric α is invariant under any conformal change of $(\alpha,\beta)-$metric.

In [1], the conformal change of

$$B^{ij} = \frac{\alpha L_\beta}{L_\alpha}\left(s_0^i y^i - s_0^j y^i\right) + \frac{\alpha^2 L_{\alpha\alpha}}{\beta L_\alpha}(b^i y^j - b^j y^i)$$

by using

$\bar{L} = e^\sigma L(\alpha,\beta), \bar{L}_{\bar{\alpha}} = L_\alpha, \bar{L}_{\bar{\alpha}\bar{\alpha}} = e^{*\sigma} L_{\alpha\alpha}$, $\bar{L}_{\bar{\beta}} = L_\beta, \bar{\gamma}^2 = e^{2\sigma}\gamma^2$ and $\bar{C}^{\cdot} = e^\sigma(C^{\cdot} + D^{\cdot})$

where

$$D^{\cdot} = \frac{\alpha\beta\{(\beta\alpha^2 - \sigma_0\beta)L_\alpha - \alpha(b^2\sigma_0 - \rho\beta)L_\beta\}}{2(\beta^2 L_\alpha + \alpha\gamma^2 L_{\alpha\alpha})}$$

Hence the conformal change B^{ij} can be written as

$$\bar{B}^{ij} = \frac{\alpha L_\beta}{L_\alpha}\left(s_0^i y^i - s_0^j y^i\right) + \frac{\alpha^2 L_{\alpha\alpha}}{\beta L_\alpha}(b^i y^j - b^j y^i) + \left(\frac{\alpha\sigma_0 L_\beta}{L_\alpha} + \frac{\alpha^2 L_{\alpha\alpha}}{\beta L_\alpha} D^{\cdot}\right)(b^i y^j - b^j y^i) - \frac{\alpha\beta L_\beta}{2L_\alpha}(\sigma^i y^j - \sigma^j y^i) = B^{ij} + C^{ij} \quad (3.2)$$

where

$$C^{ij} = \frac{\alpha^2 L_{\alpha\alpha}}{\beta L_\alpha}(b^i y^j - b^j y^i) + \left(\frac{\alpha\sigma_0 L_\beta}{L_\alpha} + \frac{\alpha^2 L_{\alpha\alpha}}{\beta L_\alpha} D^{\cdot}\right)(b^i y^j - b^j y^i) - \frac{\alpha\beta L_\beta}{2L_\alpha} \quad (3.3)$$

and hence

$$\bar{\Omega} = e^{2\sigma}\Omega, \bar{A} = e^{*\sigma} A, \bar{B} = e^{2\sigma} B$$

Now we apply conformal transformation to B_m^{im} and get

$$\bar{B}_m^{im} = B_m^{im} + K_m^{im} \quad (3.4)$$

where

$$K_m^{im} = \frac{(n+1)\alpha L_\beta}{L_\alpha}(\sigma_0 b^i - \beta\sigma^i) + \frac{(n+1)\alpha^3 \Omega L_{\alpha\alpha} b^i + \alpha\beta\gamma^2 A\gamma^i}{\Omega^2}(\rho\alpha^2 - \sigma_0\beta) + \left[\frac{\alpha^2(n+1)\Omega L_\beta L_{\alpha\alpha} b^i + B y^i}{L_\alpha \Omega^2} - \frac{\alpha^3 L_{\alpha\alpha} y^i}{\Omega}\right](b^2\sigma_0 - \rho\beta) \quad (3.5)$$

Hence, we have the following

Theorem 3.2.:- The necessary and sufficient condition for a Douglas space of second kind with $(\alpha,\beta)-$ metric to be a Douglas space of second kind under conformal transformation, is that $K_m^{im}(x)$ are homogenous polynomial in (y^m) of degree two.

4. CONFORMAL CHANGE OF DOUGLAS SPACE OF SECOND KIND WITH CERTAIN $(\alpha,\beta)-$METRIC:-

The important examples of Finsler space with $(\alpha,\beta)-$metric are Randers space, Kropina space, generalized Kropina space and Matsumoto space. The conditions for Douglas space of second kind with Randers metric and Kropina metric to be a Douglas space of second kind under conformal change have been obtained in [15].

In this section, we extend the study on conformal change of Douglas space of second kind and obtain

the conditions for Douglas space of second kind with special $(\alpha,\beta)-$ metric $\mathrm{L} = \beta + \frac{\beta^2}{\alpha}$.
to be a Douglas space of second kind under conformal change.

5. CONFORMAL CHANGE OF DOUGLAS SPACE OF SECOND KIND WITH A SPECIAL $(\boldsymbol{\alpha},\boldsymbol{\beta})-$ METRIC, $\mathbf{L} = \boldsymbol{\beta} + \frac{\boldsymbol{\beta}^2}{\boldsymbol{\alpha}}$

Let us consider the Douglas space of second kind with special (α,β) −metric $L = \beta + \frac{\beta^2}{\alpha}$. Then we have

$$\left.\begin{array}{c} L_\alpha = \dfrac{-\beta^2}{\alpha^2}, L_{\alpha\alpha} = \dfrac{2\beta^2}{\alpha^3}, \quad L_{\alpha\alpha\alpha} = \dfrac{-6\beta^2}{\alpha^4} \\ L_\beta = 1 + \dfrac{2\beta}{\alpha},\ L_{\beta\beta} = \dfrac{2}{\alpha}, L_{\alpha\beta} = -\dfrac{2\beta}{\alpha^2} \\ \Omega = \dfrac{\beta^2}{\alpha^2}(\beta^2 + 2\alpha^2 b^2) \end{array}\right\} \quad (5.1)$$

Substituting (5.1) into (2.5), we have

$$\left.\begin{array}{c} A = -6\dfrac{\beta^4}{\alpha^5} \\ B = \dfrac{2(\alpha+2\beta)\beta^4}{\alpha^6}\begin{Bmatrix} 3\beta b^2\alpha^2 - 3\beta^3 - 3\beta b^2 \\ +12\beta^2 - 8b^2\alpha^2 \end{Bmatrix} \end{array}\right\} (5.2)$$

Further substituting (5.1) and (5.2) into (2.4), we get

$$2K^{im}{}_m = \frac{(n+1)(\alpha+2\beta)\alpha^2}{\beta^2}(\sigma_0 b^i - \beta\sigma^i) + \frac{\begin{Bmatrix} 2(n+1)(\beta^2+2\alpha^2b^2)\alpha^2\beta^4 b^i \\ -6\beta^5(b^2\alpha^2-\beta^2)y^i \end{Bmatrix}}{\alpha^2\beta^2(\beta^2+2\alpha^2b^2)}(\rho\alpha^2 - \sigma_0\beta) + \frac{(\alpha+2\beta)\begin{Bmatrix} 2(n+1)\alpha^2(\beta^2+2\alpha^2b^2)b^i + \\ 2\begin{pmatrix} 3\beta b^2\alpha^2 - 3\beta^3 \\ -3\beta b^2 + 12\beta^2 - 8b^2\alpha^2 \end{pmatrix} y^i \end{Bmatrix}}{\beta^2(\beta^2+2\alpha^2b^2)^2}(b^2\sigma_0 - \rho\beta) - \frac{2\alpha^2 y^i}{(\beta^2+2\alpha^2b^2)}(b^2\sigma_0 - \rho\beta) \qquad (5.3)$$

$$2\alpha^2\beta^2(\beta^2+2\alpha^2b^2)^2 K^{im}{}_m = 2(n+1)(\alpha+2\beta)\alpha^2(\beta^2+2\alpha^2b^2)^2(\sigma_0 b^i - \beta\sigma^i) + \begin{Bmatrix} \alpha^2(n+1)(\beta^2+2\alpha^2b^2)b^i \\ -3\beta(b^2\alpha^2-\beta^2)y^i \end{Bmatrix} 2(\rho\alpha^2 - \sigma_0\beta) - 2(\beta^2+2\alpha^2b^2)\beta^2\alpha^2 y^i(b^2\sigma_0 - \rho\beta) + \begin{Bmatrix} 2\alpha^2(n+1)(\beta^2+2\alpha^2b^2)(\alpha+2\beta)b^i + \\ 2(\alpha+2\beta)\begin{pmatrix} 3\beta b^2\alpha^2 - 3\beta^3 - 3\beta b^2 \\ +12\beta^2 - 8b^2\alpha^2 \end{pmatrix} y^i \end{Bmatrix}(b^2\sigma_0 - \rho\beta)\ . \qquad (5.4)$$

Hence $K^{im}_m(x)$ are homogenous polynomial in (y^m) of degree two.

Theorem 5.1. The Douglas space of second kind with special $(\alpha,\beta)-$ metric is conformally transformed to a Douglas space of second kind.

CONCLUSION: - Douglas space with (α,β)-metric have been treated by some authors [1], [17]. Conformal change is one of the important transforms which preserves the angle and this theory is developed in 1929 and studied by many author.

In this paper, we found the conditions for a Finsler space with special (α,β)- metric $\mathrm{L} = \beta + \frac{\beta^2}{\alpha}$ to be a Douglas space. Further, we found the conditions for a conformally transformed Douglas space with the above mentioned special (α,β)- metric to be a Douglas space

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